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EFFECTS OF VISCOUS DISSIPATION AND HEAT GENERATION (ABSORPTION) IN A THERMAL BOUNDARY LAYER OF A NON-NEWTONIAN FLUID OVER A CONTINUOUSLY MOVING PERMEABLE FLAT PLATE

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The problem of boundary-layer flow and heat transfer of a non-Newtonian power-law fluid over a moving porous infinite flat plate in the presence of viscous dissipation and heat generation or absorption is investigated analytically. It is assumed that both the momentum and the energy equations are coupled by the stress friction factor, and an assumption is introduced regarding the heat-transfer index. It is found that exact analytical solutions for velocity and temperature exist only for pseudoplastic fluids in the presence of suction at the surface. The effects of the suction parameter, Eckert number, and the heat generation or absorption parameter on the velocity and temperature profiles, as well as on the skin-friction coefficient and Nusselt number are discussed.

Key words: *non-Newtonian power-law fluids, moving surface, heat generation or absorption.*

Introduction. Flow and heat transfer associated with a continuously moving surface in an otherwise quiescent fluid is of relevance in many manufacturing processes, such as hot rolling, wire drawing, continuous casting, paper production, and extrusion of metallic, glass, and polymeric materials. Sakiadis [1, 2] initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al. [3] extended Sakiadis' problem to the case in which the transverse velocity at the moving surface is non-zero, with heat and mass transfer in the boundary layer being taken into account. Tsou et al. [4] confirmed the results of Sakiadis experimentally and investigated the heat-transfer effects of a moving sheet with constant surface velocity and temperature. Chen [5] studied the problem of forced convection flow and heat transfer about a flat sheet with suction or injection, continuously moving in a quiescent or flowing fluid. The thermal boundary-layer problem of a semi-infinite flat plate moving in a constant-velocity free stream was studied by Fang [6]. Cortell [7] examined numerically the momentum and heat transfer of an incompressible viscous moving fluid over a moving flat surface.

All the above-mentioned investigations were restricted to flows of a Newtonian fluid. Many important fluids, however, such as molten plastics, polymers, printing ink, food stuff, etc., are non-Newtonian in their flow characteristics. The interest in studying flow and heat-transfer characteristics of non-Newtonian fluids has increased in the last four decades because of their importance in many industries. Schowalter [8] was the first one who studied the boundary-layer flow of a non-Newtonian fluid. Similarity solutions were obtained by Acrivos et al. [9], Kapur and Srivastava [10], Lee and Ames [11], Berkoveskii [12], Hansen and Na [13], and others. Thomson and Snyder [14–15] studied the effect of injection on the flow of a non-Newtonian power-law fluid over a flat plate. Liu [16] presented a class of asymptotic solutions for the flow of power-law fluids over a flat plate with suction.

Forced convective flow over a flat plate in non-Newtonian power-law fluids was studied by Huang and Lin [17]. Howell et al. [18] applied the Merk–Chao series expansion method to solve the problem of combined momentum and heat transfer in the boundary layer of the moving surface in a power-law fluid. Rao et al. [19] studied the problem of momentum and heat transfer in a power-law fluid with arbitrary injection (suction) at a moving wall using the

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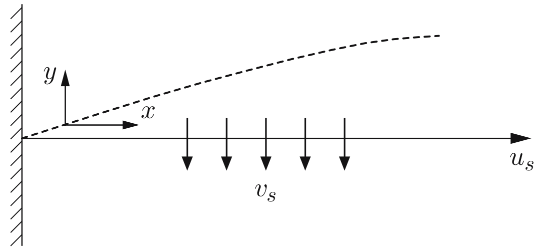

Fig. 1. Flow geometry and coordinate system.

Merk-Chao series. Momentum and heat transfer from a continuous moving surface to a power-law fluid was studied by Sahu et al. [20]. Ariel [21] studied the problem of non-Newtonian power-law fluids over a continuously stretching surface. Gupta et al. [22] analyzed the problem of steady flow of a non-Newtonian fluid past an infinite porous flat plate subject to suction. Cortell [23] analyzed the flow of an electrically conducting power-law fluid in the presence of a uniform transverse magnetic field over a stretching sheet. Mahmoud and Mahmoud [24] studied the laminar boundary-layer flow over a continuously moving surface in a non-Newtonian power-law fluid in the presence of a transverse magnetic field analytically, using the successive approximations method. Mahmoud and Megahed [25] presented the problem of flow and heat transfer of an electrically conducting non-Newtonian power-law fluid with low electrical conductivity on a continuously moving infinite porous plate in the presence of a uniform magnetic field. The problem of boundary-layer flow and heat transfer of a non-Newtonian power-law fluid over a continuously moving permeable surface in the presence of viscous dissipation and heat generation or absorption has not been considered within available literature.

The aim of the present work is to study the effects of the suction parameter and the heat generation or absorption parameter on the velocity and temperature profiles, as well as on the skin-friction coefficient and the Nusselt number.

1. Formulation of the Problem. We consider a flat plate, which is emerged from a slot at a fixed temperature T_s and is continuously moving with a steady velocity u_s in an otherwise quiescent non-Newtonian power-law fluid with a temperature T_∞. The physical model and the coordinate system are shown in Fig. 1. The origin is located at the slot; x and y are Cartesian coordinates along and normal to the surface, respectively.

We assume that the fluid has the following transport properties [26–33]:

$$\tau_{ij} = -P\delta_{ij} + K|I_2/2|^{(n-1)/2} e_{ij}; \qquad (1)$$

$$q = -\varkappa |I_2/2|^{(n-1)/2} \operatorname{grad} T.$$

Here τ_{ij} and e_{ij} are the stress and strain-rate tensors, δ_{ij} is the unit tensor, I_2 is the second invariant of the strain-rate tensor, P and T are the pressure and temperature of the fluid, respectively, q is the heat flux, $\varkappa$ is the modified thermal conductivity, $K > 0$ is the fluid consistency index, and $n > 0$ is a rheological index known as the power-law index. When $n = 1$, Eq. (1) reduces to a constitutive equation for a viscous Newtonian fluid with $K = \mu$ (μ is the coefficient of viscosity). Fluids with $n < 1$ are called pseudo-plastic fluids and fluids with $n > 1$ are called dilatant fluids.

The governing equations for the problem taking into account the viscous dissipation and the heat generation or absorption effects in the energy equation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{K}{\rho}\,\frac{\partial}{\partial y}\Big(\Big|\frac{\partial u}{\partial y}\Big|^{n-1}\frac{\partial u}{\partial y}\Big),$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial}{\partial y}\Big(\Big|\frac{\partial u}{\partial y}\Big|^{n-1}\frac{\partial T}{\partial y}\Big) + \frac{K}{\rho c_p}\Big(\Big|\frac{\partial u}{\partial y}\Big|^{n-1}\Big(\frac{\partial u}{\partial y}\Big)^2\Big) + \frac{Q}{\rho c_p},$$

where u and v are the velocity components in the x and y directions, respectively, ρ is the fluid density, Q is the internal heat generation or absorption rate per unit volume, $\alpha = \varkappa/(\rho c_p)$ is the thermal diffusivity, and c_p is the specific heat at constant pressure. Since the plate is considered to be of infinite extent, all derivatives with respect to x vanish:

$$\frac{dv}{dy} = 0; \tag{2}$$

$$\begin{gathered} K\,\frac{d}{dy}\Big(\Big|\frac{du}{dy}\Big|^{n-1}\frac{du}{dy}\Big) - v\,\frac{du}{dy} = 0, \\ \alpha\,\frac{d}{dy}\Big(\Big|\frac{du}{dy}\Big|^{n-1}\frac{dT}{dy}\Big) - v\,\frac{dT}{dy} + \frac{K}{\rho c_p}\Big(\Big|\frac{du}{dy}\Big|^{n-1}\Big(\frac{du}{dy}\Big)^2\Big) + \frac{Q}{\rho c_p} = 0. \end{gathered} \tag{3}$$

The boundary conditions for Eqs. (2), (3) are

$$y = 0\colon \quad u = u_s, \quad T = T_s, \qquad y \to \infty\colon \quad u \to 0, \quad T = T_\infty.$$

The solution of Eq. (2) is

$$v = -v_s, \tag{4}$$

where $v_s > 0$ is the constant normal velocity at the plate.

The following dimensionless variables are introduced:

$$f = \frac{u}{u_s}, \quad c = \frac{v_s}{u_s}, \quad \theta = \frac{T}{T_\infty}, \quad \eta = \frac{\rho^{1/n} u_s^{(2-n)/n}}{K^{1/n}}\, y. \tag{5}$$

The dependence of the internal heat generation or absorption rate on the space coordinate can be taken in the form

$$Q = Q_0\, \mathrm{e}^{-\eta}\,. \tag{6}$$

Substituting Eqs. (4)–(6) into Eqs. (3) yields the following dimensionless governing equations:

$$\frac{d}{d\eta}\Big(-\frac{df}{d\eta}\Big)^n - c\,\frac{df}{d\eta} = 0; \tag{7}$$

$$\frac{d^2\theta}{d\eta^2}\Big(-\frac{df}{d\eta}\Big)^{n-1} + \Big((1-n)\Big(-\frac{df}{d\eta}\Big)^{n-2}\frac{d^2 f}{d\eta^2} + c\,\mathrm{Pr}\Big)\frac{d\theta}{d\eta} + \mathrm{Ec}\,\mathrm{Pr}\,\Big(-\frac{df}{d\eta}\Big)^{n+1} + \gamma\, \mathrm{e}^{-\eta} = 0. \tag{8}$$

Here $\mathrm{Pr} = \mu c_p/\varkappa$ is the non-Newtonian Prandtl number, $\mathrm{Ec} = u_s^2/(c_p T_\infty)$ is the Eckert number, c is the constant suction parameter, and $\gamma = Q_0 K^{(n+1)/n}/(\varkappa T_\infty \rho^{(n+1)/n} u_s^{2/n})$ is the heat generation ($\gamma > 0$) or absorption ($\gamma < 0$) parameter.

The dimensionless boundary conditions for Eqs. (7), (8) are

$$\eta = 0\colon \quad f = 1, \quad \theta = \theta_r, \qquad \eta \to \infty\colon \quad f \to 0, \quad \theta \to 1, \tag{9}$$

where $\theta_r = T_s/T_\infty$.

The surface shear stress is defined as

$$\tau_s = \Big(K\Big|\frac{du}{dy}\Big|^{n-1}\frac{du}{dy}\Big)_{y=0}.$$

In terms of the dimensionless variables, the skin-friction coefficient is given by

$$C_f = 2\tau_s/(\rho u_s^2) = 2|f'(0)|^n. \tag{10}$$

The rate of heat transfer at the plate is

$$q_s = -\varkappa\,\frac{dT}{dy}\Big|_{y=0},$$

and the Nusselt number is determined by the expression

$$\mathrm{Nu} = -\theta'(0).$$

2. Exact Solutions for the Momentum and Energy Equations. Integrating Eq. (7) and using the boundary conditions (9), we obtain

$$f = (a(\eta + c_2))^{n/(n-1)}/c, \tag{11}$$

where $c_2 = n/((1-n)c^{1/n})$ and $a = c(1-n)/n$ $(0 < n < 1)$.

TABLE 1

Values of $-\theta'(0)$ and $|f'(0)|^n$ for Various Values of n, c, Pr, γ, and Ec

n	c	Ec	Pr	γ	$\lvert f'(0)\rvert^n$	$-\theta'(0)$
0.6	2.0	0.1	3	0.5	2.0	8.2545
0.7	2.0	0.1	3	0.5	2.0	6.9987
0.8	2.0	0.1	3	0.5	2.0	6.1839
0.9	2.0	0.1	3	0.5	2.0	5.6167
0.8	0.5	0.1	3	0.5	0.5	0.7778
0.8	1.0	0.1	3	0.5	1.0	2.3500
0.8	1.5	0.1	3	0.5	1.5	4.1777
0.8	2.0	0.1	3	0.5	2.0	6.1839
0.8	1.0	0.1	3	0.5	1.0	2.3500
0.8	1.0	0.3	3	0.5	1.0	2.0500
0.8	1.0	0.5	3	0.5	1.0	1.7500
0.8	1.0	0.1	1	0.5	1.0	0.4500
0.8	1.0	0.1	3	0.5	1.0	2.3500
0.8	1.0	0.1	7	0.5	1.0	6.1466
0.8	1.0	0.5	3	−0.3	1.0	2.5500
0.8	1.0	0.5	3	−0.2	1.0	2.4500
0.8	1.0	0.5	3	0	1.0	2.2500
0.8	1.0	0.5	3	0.5	1.0	1.7500
0.8	1.0	0.5	3	1.0	1.0	1.2500
0.8	1.0	0.5	3	2.0	1.0	0.2500

Substituting Eq. (11) into Eq. (8), we obtain the following final form of the energy equation:

$$\frac{d^2\theta}{d\eta^2}\,(a(\eta+c_2)) + (a + c\,\mathrm{Pr})\,\frac{d\theta}{d\eta} + \mathrm{Ec}\,\mathrm{Pr}(a(\eta+c_2))^{(n+1)/(n-1)} + \gamma\,\mathrm{e}^{-\eta} = 0. \tag{12}$$

Integrating Eq. (12) under the boundary conditions (9) with $\theta_r = 2$, we obtain

$$\begin{aligned}\theta = \Big\{&(\eta+c_2)^{-p/a}\Big[a\Big(4an^2(c_2^{p/a} + (\eta+c_2)^{p/a}) + (n-1)\{2c_2^{p/a}np \\ &+ c_2^{(a+p)/a}(ac_2)^{(n+1)/(n-1)}(n-1)b - c_2(n-1)b(\eta+c_2)^{p/a}(a(\eta+c_2))^{(n+1)/(n-1)} \\ &+ (\eta+c_2)^{p/a}[2np + b\eta(a(\eta+c_2))^{(n+1)/(n-1)} - nb\eta(a(\eta+c_2))^{(n+1)/(n-1)}]\}\Big) \\ &+ 2n\,\mathrm{e}^{c_2}(2na + (n-1)p)\gamma\Gamma(p/a, c_2) - 2n\,\mathrm{e}^{c_2}(2na + (n-1)p)\gamma\Gamma(p/a, \eta+c_2)\Big]\Big\}/[2an(2an + (n-1)p)],\end{aligned}$$

where $p = c\,\mathrm{Pr}$, $b = \mathrm{Pr}\,\mathrm{Ec}$, and Γ is the gamma function.

3. Results and Discussion. Examples of the velocity and temperature distributions for various values of c, n, Pr, γ, Ec, and $\theta_r = 2$ are shown in Figs. 2–6. Figures 2 and 3 display the effect of the suction parameter c on the velocity and temperature distributions at $n = 0.8$. It is seen from these figures that imposition of the wall suction has the tendency to reduce both the hydrodynamic and thermal boundary layers, resulting in a decrease in fluid velocity and temperature. Figure 4 depicts the effect of the non-Newtonian Prandtl number Pr on the temperature profiles. It is evident from this figure that the temperature decreases as Pr increases. This is in agreement with the physical fact that the thermal boundary-layer thickness decreases with increasing Pr. Figure 5 displays the temperature as a function of the Eckert number Ec. It is clear from this figure that the temperature of the fluid is at a higher level when viscous dissipation is considered than when it is neglected. Figure 6 illustrates the influence of the heat generation or absorption parameter γ on the temperature profiles. In the case of heat generation ($\gamma > 0$), an increase in the values of γ has a tendency to increase the temperature, while the temperature decreases with increasing absolute value of γ in the case of heat absorption ($\gamma < 0$). Table 1 illustrates the effects of the power-law index n, the suction parameter c, the Eckert number Ec, the heat generation or absorption parameter γ, and the Prandtl number Pr on the skin-friction coefficient C_f and the Nusselt number Nu. It follows from Eqs. (10) and (11) that the skin-friction coefficient depends only on the suction parameter. As is shown in Table 1, an increase in the suction parameter leads to an increase in the skin-friction coefficient. It is also observed from Table 1 that

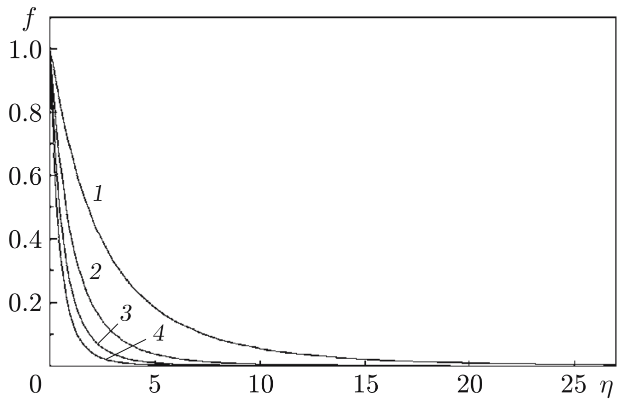

Fig. 2

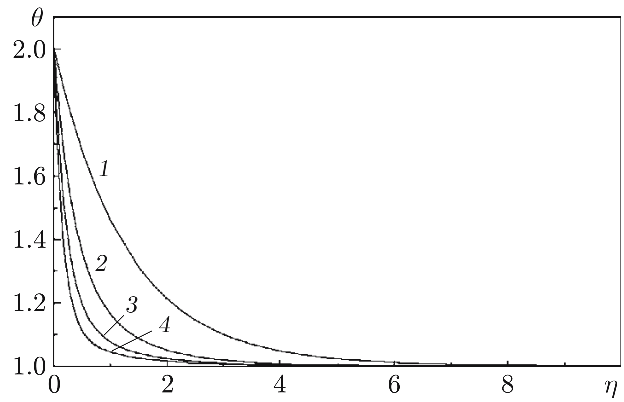

Fig. 3

Fig. 2. Velocity distributions for $n = 0.8$ and $c = 0.5$ (1), 1 (2), 1.5 (3), and 2 (4).

Fig. 3. Temperature distributions for $n = 0.8$, $\gamma = 0.5$, $\mathrm{Pr} = 3$, $\mathrm{Ec} = 0.1$, and $c = 0.5$ (1), 1 (2), 1.5 (3), and 2 (4).

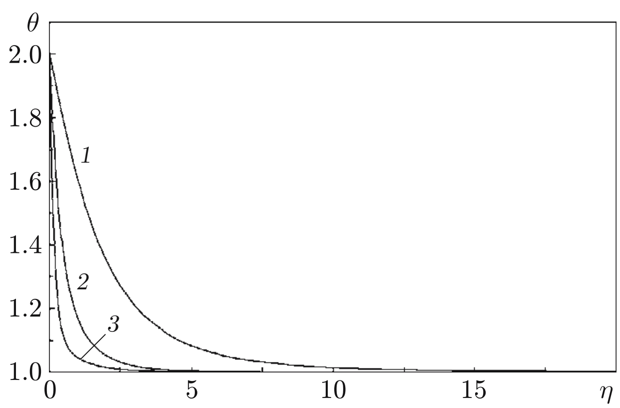

Fig. 4

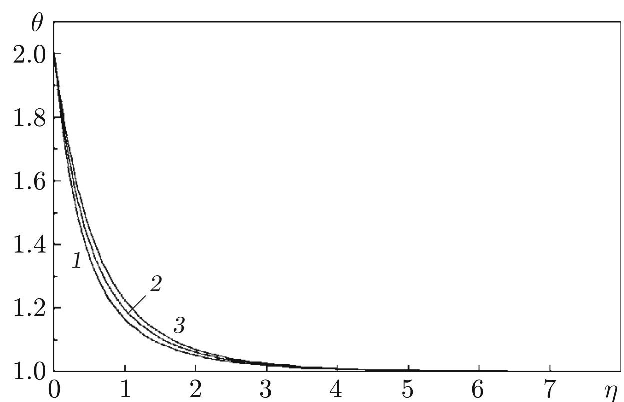

Fig. 5

Fig. 4. Temperature distributions for $n = 0.8$, $\gamma = 0.5$, $c = 1$, $\mathrm{Ec} = 0.1$, and $\mathrm{Pr} = 1$ (1), 3 (2), and 7 (3).

Fig. 5. Temperature distributions for $n = 0.8$, $\gamma = 0.5$, $\mathrm{Pr} = 3$, $c = 1$, and $\mathrm{Ec} = 0.1$ (1), 0.3 (2), and 0.5 (3).

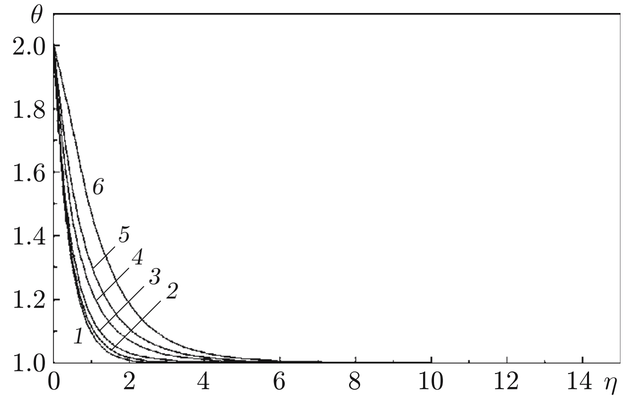

Fig. 6. Temperature distributions for $n = 0.8$, Ec = 0.5, Pr = 3, $c = 1$, and $\gamma = -0.3$ (1), 0.2 (2), 0 (3), 0.5 (4), 1 (5), and 2 (6).

the Nusselt number decreases with increasing n and increases with increasing Prandtl number. This is because a fluid with a higher value of Pr possesses a larger heat capacity and, hence, intensifies the heat transfer. Therefore, cooling of the heated surface can be improved by choosing a coolant having a larger Prandtl number. Also, as the suction parameter increases, the Nusselt number increases. In the case $\gamma > 0$, the Nusselt number decreases with increasing γ; in the case with $\gamma < 0$, vice versa, the Nusselt number increases with increasing $|\gamma|$. An increase in the Eckert number, however, reduces the Nusselt number, because the effectiveness of plate cooling decreases with increasing Eckert number, which is the measure of heat produced by friction. This behavior is consistent with the dimensionless temperature profiles shown in Fig. 5.

Conclusions. An exact solution of the problem of boundary-layer flow of a non-Newtonian power-law fluid over a continuously moving infinite permeable plate with heat generation or absorption is obtained. The solution exists only for pseudo-plastic fluids ($n < 1$), provided that there is suction at the plate. The skin-friction coefficient and the Nusselt number are found to increase as the suction parameter increases. It is found, however, that the Nusselt number increases as either the Prandtl number or the heat absorption parameter increases, while it decreases as the Eckert number, the power-law index, or the heat generation parameter increases.

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